GEORGIA INSTITUTE OF TECHNOLOGY — OFFICE OF CONTRACT ADMINISTRATION

PROJECT ADMINISTRATION DATA SHEET

[x] ORIGINAL [ ] REVISION NO. ______

Project No. E-21-E48 (Subproject under E-21-E00/ Paris/EE) — DATE 3/24/82

Project Director: K. R. Davey — School/~~Lab~~ Electrical Eng.

Sponsor: Naval Coastal Systems Center, Panama City, FL 32407

Type Agreement: Contract N00612-79-D-8004, Delivery Order No. HR-48

Award Period: From 3/12/82 To 12/1/82 (Performance) ______ (Reports)

Sponsor Amount: $16,396 — Contracted through:

Cost Sharing: None — GTRI/~~GIT~~

Title: The Determination of 3-Dimensional Magnetic Fields in Conducting Media

ADMINISTRATIVE DATA

OCA Contact: William F. Brown x4820

1) Sponsor Technical Contact:

2) Sponsor Admin/Contractual Matters:
Office of Naval Research
Resident Representative
206 O'Keefe Building
Georgia Institute of Technology
Atlanta, Georgia 30332

Defense Priority Rating: DO-C9

Security Classification: unclassified

RESTRICTIONS

See Attached Gov't Contract Supplemental Information Sheet for Additional Requirements.

Travel: Foreign travel must have prior approval — Contact OCA in each case. Domestic travel requires sponsor approval where total will exceed greater of $500 or 125% of approved proposal budget category.

Equipment: Title vests with Government; except that items costing less than $1K vest with GIT upon acquisition if prior approval to purchase is obtained from Contracting Officer.

COMMENTS:

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FORM OCA 4:781

GEORGIA INSTITUTE OF TECHNOLOGY OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION SHEET

Date 3/16/83

Project Title: The Determination of 3-Dimensional Magnetic Fields in Conducting Media

Project No: E-21-E48 (subproject under E-21-E00/Paris/EE)

Project Director: K. R. Davey

Sponsor: Naval Coastal Systems Center, Panama City, FL 32407

Effective Termination Date: 12/1/82

Clearance of Accounting Charges: 12/1/82

Grant/Contract Closeout Actions Remaining:

- [x] Final Invoice ~~and Closing Documents~~
- [ ] Final Fiscal Report
- [x] Final Report of Inventions
- [x] Govt. Property Inventory & Related Certificate
- [ ] Classified Material Certificate
- [ ] Other ______

Assigned to: Elect. Engr. (School/Laboratory)

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FORM OCA 10:781

BIMONTHLY STATUS REPORT

NAVAL COASTAL SYSTEMS CENTER
OMNIBUS R&D PROGRAM
CONTRACT NO. N00612-79-C-8004

Period Covered: April 1 - May 31

Order Number: HR- 48 Title: 3-D Magnetic Fields in Conducting Media

Task Leader: Dr. Kent R. Davey

Institution: Georgia Institute of Technology

A. SUMMARY STATEMENT OF WORK COMPLETED DURING THE PAST TWO MONTHS

Completion of the theoretical T-Ω analysis for the wedge problem has been accomplished. The advantages of this method in solving the generalized problem over existing vector potential solutions has been considered; reduction in matrix handling from a 12x12 to a 2x2 is seen. A paper has been written and submitted to IEEE Magnetics illustrating the technique and concepts.

B. WORK SCHEDULE STATUS

The work schedule has been closely adhered to at present.

C. BRIEF STATEMENT OF PLANNED WORK FOR THE NEXT TWO MONTHS

Numerical formulation of the problem for the horizontal dipole in the half plane conducting space is next on the schedule. Emphasis must be put on efficiently handling the large matrices expected.

D. PROBLEM AREAS

One problem area is expected from sources extending through boundaries. It is speculated that point current density sources may best be handled by imposing an additional constraint on the loop integral of tangential $\bar{T}$.

E. FUNDS EXPENDED

To Date:

This Two Month Period:

Funds Remaining:

Percent of Funds Expended:

Percent of Task Completed:

BIMONTHLY STATUS REPORT

NAVAL COASTAL SYSTEMS CENTER
OMNIBUS R&D PROGRAM
CONTRACT NO. N00612-79-C-8004

Period Covered: June 1 to July 31, 1982

Order Number: HR- 48 Title: Determination of 3-D Magnetic Fields in Conducting Media"

Task Leader: Kent Davey

Institution: Georgia Institute of Technology

A. SUMMARY STATEMENT OF WORK COMPLETED DURING THE PAST TWO MONTHS

The last two months have virtually focused on checking the analytical and numerical formulations developed. We're now convinced that the analytic solutions of Banister, et. al., are correct. The numerical formulation fails as yet to converge on this result. Much effort has been spent checking the technique and isolating errors.

B. WORK SCHEDULE STATUS

We are behind schedule due to failure of numerical convergence. Progress should move quickly once this difficulty is cleared.

C. BRIEF STATEMENT OF PLANNED WORK FOR THE NEXT TWO MONTHS

The immediate objective is to realize a convergence of the numerical technique. Next, the horizontal dipole will be implemented and compared to exact solutions. Finally the 3-D method applied to the wedge will be outlined.

D. PROBLEM AREAS

The problem area is basically one of determining how best to get an accurate integral representation over an unbounded surface.

E. FUNDS EXPENDED

To Date: $11,518.28

This Two Month Period: -0-

Funds Remaining: $4,877.72

Percent of Funds Expended: 70%

Percent of Task Completed: 44%

BIMONTHLY STATUS REPORT

NAVAL COASTAL SYSTEMS CENTER
OMNIBUS R&D PROGRAM
CONTRACT NO. N00612-79-C-8004

Period Covered: August 1 - September 30

Order Number: HR- 48 Title: "Calculation of Magnetic Fields in Conducting Media"

Task Leader: Dr. K. R. Davey

Institution: Georgia Institute of Technology

A. SUMMARY STATEMENT OF WORK COMPLETED DURING THE PAST TWO MONTHS

Demonstration of the integral technique convergence for the vertical dipole in a half space conducting medium has been achieved. The Green's function appropriate for the current dipoles is better understood in the light of work by Mittra. Appropriate application of the technique is now being attempted for the horizontal dipole. Geometric symmetries are being sought to keep the problem manageable. Appropriate use of the gauge condition for the horizontal dipole has just been realized.

B. WORK SCHEDULE STATUS

We are at present a bit lagging behind anticipated progress. Solution convergence for the vertical dipole was delayed due to usage of an inappropriate Green's function.

C. BRIEF STATEMENT OF PLANNED WORK FOR THE NEXT TWO MONTHS

Emphasis will be placed on accurate prediction of fields for the horizontal dipole. Foundation work for the arbitrary discretization of the surface for integration must be sought for field determination in the generalized cases.

D. PROBLEM AREAS

Immediate problem areas are lack of convergence of the horizontal case and discretization in the generalized case.

E. FUNDS EXPENDED

To Date: 4/7 of all funds expended to date. $11,518.28

This Two Month Period: -0-

Funds Remaining: $4,877.72

Percent of Funds Expended: 70%

Percent of Task Completed: 44%

E21-648

NAVAL COASTAL SYSTEMS CENTER
OMNIBUS R&D PROGRAM
CONTRACT NO. N00612-79-C-8004

Period Covered: October 1 - November 30, 1982

Order Number: HR-48 Title: 3-Dimensional Magnetic Field Calculations via the Fredholm Integral

Task Leader: Dr. Kent R. Davey

Institution: Georgia Institute of Technology

A. SUMMARY STATEMENT OF WORK COMPLETED DURING THE PAST TWO MONTHS

Two papers discussing the techniques application to the time varying vertical and horizontal dipole in a conducting half space have been completed. This work describes the successful predictions of the technique and verifies the theories predicted thus far. Successful prediction of lateral waves with the integral technique confirms its usefulness at high frequencies. Both papers are being sent for publication to IEEE Magnetics. In addition, a Tech note is being processed internally to relay the information now known.

B. WORK SCHEDULE STATUS

Adequate progress is being realized presently.

C. BRIEF STATEMENT OF PLANNED WORK FOR THE NEXT TWO MONTHS

The next two months will focus on predicting the fields for the arbitrary dipole and the wedge. The latter problem will require fabrication of an experimental prototype for theoretical verification.

D. PROBLEM AREAS

The wedge presents a major problem in terms of numerical complexity. Emphasis must be given to optimal usage of basic functions over surface elements in evaluating the Fredholm Integrals.

E. FUNDS EXPENDED

To Date: 2 2/3 man months expended.

This Two Month Period:

Funds Remaining:

Percent of Funds Expended:

Percent of Task Completed:

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL ENGINEERING
ATLANTA, GEORGIA 30332

TELEPHONE: (404) 894- 2961

January 31, 1983

Naval Coastal Systems Center
Panama City, Florida 32407

Ref: Navy Contract No. N00612-79-C-8004, Task HR-48, Georgia Institute of Technology, Final Report, Period Covered - 3/12/82 - 12/1/82

Dear Sir:

Enclosed is the Final Report entitled "The Determination of Three Dimensional Magnetic Fields in Conducting Media" which was prepared by Dr. K. R. Davey of the Georgia Institute of Technology.

The report is submitted as per contract specifications.

Thank you.

Sincerely,

Marsha Segraves
Admin. Asst.

cc: Tom Bryant, ONR
OCA (2)

/ms

Final Report for

Naval Coastal Systems Center

Contract HR-48

The Determination of Three Dimensional Magnetic Fields in Conducting Media

by

Dr. Kent R. Davey
School of Electrical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

January 20, 1983

ABSTRACT

The solution of eddy current field problems in geometries with unbounded surfaces is generally quite difficult. Any discretization of volume space as suggested in a finite element or finite difference technique is ruled out as numerically unrealizable. Integral techniques allow an equivalent discretization of a surface with the added advantage of reducing the number of unknowns from N^3 to N^2. Representation of the magnetic field intensity as a vector plus the gradient of a scalar (T-Ω representation) lends itself to the integral formulation and to the matching of boundary conditions. Discussed herein is the application of the T-Ω method to the problem of time varying vertical and horizontal dipoles imbedded in a conducting infinite half space. The theoretical formulation is laid out and the numerical results are compared to known exact solutions via Sommerfeld integral equations. Also, the accuracy of the integral technique at high frequencies is investigated with the results compared to analytical surface wave expressions.

I. INTRODUCTION

Although the solution of 3-dimensional magnetic field problems has been considered using finite element and finite difference techniques [6,7,9,14], their usefulness is limited if the problem is truly unbounded since the number of unknowns can easily become unmanageable. Integral techniques lessen the number of unknowns by a power of 2/3 while still retaining some degree of flexibility [5,7,8,15]. Three-dimensional solutions, however, via the magnetic vector potential, necessarily involve cross derivatives which lead to difficult matching of boundary conditions [8]. One way to avoid such a problem is to represent the H-field as the sum of a vector $\bar{T}$ and the gradient of a scalar $\nabla\Omega$.

By way of demonstrating the usefulness of such a technique, the authors have applied it to an unbounded problem having a known solution. The problem is shown in Figure 1. A time varying vertical and horizontal dipole respectively are submerged a depth h in a conducting half space. The source is here assumed to be a current dipole having a sinusoidal time dependence ($\cos(\omega t)$). The objective is to predict the magnetic fields both in the air and in the conducting medium, and then compare these solutions with the Sommerfeld integral solutions found in the open literature [1,2,3,4,17,18].

II. THEORETICAL ANALYSIS

Consider first the general integral representation of the magnetic field as applied to the T-Ω method. We begin with the assumption that

$$\bar{H} = \bar{T} - \nabla\Omega. \tag{1}$$

Taking the curl of Ampere's Law and substituting from Faraday's Law in the sinusoidal steady state yields the expression

$$\nabla(\nabla \cdot \bar{T}) - \nabla^2\bar{T} = -\underline{\sigma}\mu j\omega(\bar{T} - \nabla\Omega) + \nabla \times \bar{J}_s \tag{2}$$

where

J_s = an imposed source current,

$\underline{\sigma} = \sigma + j\omega\varepsilon$.

With the assignment $k^2 = j\omega\mu\underline{\sigma}$, the gauge $\nabla \cdot \bar{T} - k^2\Omega = 0$, and Gauss' Law for and isotropic medium $\nabla \cdot \mu\bar{H} = 0$, two equations result

$$\nabla^2\bar{T} - k^2\bar{T} = -\nabla \times \bar{J}_s \tag{3}$$

$$\nabla^2\Omega - k^2\Omega = 0. \tag{4}$$

At this point it is necessary to introduce the Green's function appropriate for the medium, defined such that

$$\nabla^2 G - k^2 G = -\delta(x'-x,\ y'-y,\ z'-z) \tag{5}$$

Note that G is a function of primed and unprimed variable space. An expression for $\bar{T}$ in the conducting half space is obtained from multiplying (3) by G, (5) by T, subtracting the two equations, integrating the result over the conducting half space, and applying Green's integral theorem. The result is

$$T(x,y,z) = \iiint (\nabla' \times J_s') \, G_1 dv' + \oiint \left(\frac{\partial T'}{\partial n} G_1 - \frac{\partial G_1}{\partial n} T'\right) ds'. \tag{6}$$

The integration is performed over primed variable space. It is understood that only the Green's function is a function of the field point coordinates (unprimed) and hence the primed notation will be dropped. A similar expression is performed on equation (4) to give

$$\Omega_1(x,y,z) = \oiint \left(\frac{\partial \Omega_1}{\partial n} G_1 - \frac{\partial G_1}{\partial n} \Omega_1\right) ds. \tag{7}$$

The subscript on Ω and G emphasizes that the equation applies to the conducting region only. With G_2 designating the Green's function for air (with $k = \sqrt{-\omega^2 \mu_o \varepsilon_o}$) and the normal being defined as before in the positive z direction, the equivalent expression for Ω_2 is

$$\Omega_2(x,y,z) = - \oiint \left(\frac{\partial \Omega_2}{\partial n} G_2 - \frac{\partial G_2}{\partial n} \Omega_2\right) ds. \tag{8}$$

It should be noted that an assumption was made in arriving at (5) allowing for the elimination of T in region 2. The assumption is that magnetoquasistatics is valid, so that $\nabla \times \bar{H} \sim 0$, $G_2 = \frac{1}{4\pi r}$, and $\bar{T} = 0$ in the air. In cases where displacement currents are important, the generalized formulation with nonzero $\bar{T}$ in both regions must be pursued as outlined by Davey and Barnes [8].

In the magnetoquasistatic regime then, equations (6) through (8) describe the magnetic field exactly. The electric field is given by the curl of $\overline{H}$ divided by the conductivity. In cartesian coordinate systems, the source term in (6) can be written in a more useful form, i.e.,

$$\overline{T}(x,y,z) = -\iiint \overline{J} \times \nabla G \, dv + \oiint \left(\frac{\partial \overline{T}}{\partial n} G - T \frac{\partial \overline{G}}{\partial n}\right) ds. \tag{9}$$

Solution is realized by solving for the surface unknowns $\overline{T}$, $\frac{\partial \overline{T}}{\partial n}$, Ω_1, $\frac{\partial \Omega_1}{\partial n}$, $\frac{\partial \Omega_2}{\partial n}$; subject to the boundary conditions

$$\text{tangential } \overline{E} \text{ continuous} \Rightarrow \hat{n} \times \| \overline{E} \| = 0 \tag{10}$$

$$\text{tangential } \overline{H} \text{ continuous} \Rightarrow \hat{n} \times \| \overline{H} \| = 0 \tag{11}$$

$$\text{normal } \overline{B} \text{ continuous} \Rightarrow \hat{n} \cdot \| \overline{B} \| = 0 \tag{12}$$

$$\text{normal } \overline{J} \text{ continuous} \Rightarrow \hat{n} \cdot \| \overline{J} \| = 0. \tag{13}$$

In both cases being discussed, (10) turns out to be automatically satisfied and is thus of little use. Applying these conditions while letting the field point in equations (7) through (9) approach the interface yields a matrix which must be inverted for the unknowns. The final stage of the analysis is pursued separately for the vertical and horizontal cases.

III. VERTICAL DIPOLE - SOLUTION DEVELOPMENT

In the quasistatic limit the H-field in air is identically zero, so (11) implies that the H_ϕ component (the only component) is also zero on the interface. Equation (13) is automatically satisfied as is (12). Of course Ω is zero in both regions, so the total solution results from the x and y components of (9), i.e.,

$$T_x(x,y,z) = -\iiint (J_z \times \nabla G)\Big|_x \, dv + \oiint \frac{\partial T_x}{\partial n} G \, ds \tag{14}$$

or

$$T_x(x,y,z) = \frac{\partial}{\partial y'} \left(\frac{e^{-kr}}{4\pi r}\right)\Bigg|_{\substack{x'=0\\ y'=0\\ z'=0}} + \oiint \frac{\partial T_x}{\partial n} G \, ds \tag{15}$$

where

$$J_z = I\ell \, \delta(x', y', z'),$$

$$r = \sqrt{(x' - x)^2 + (y' - y)^2 + (z' - z)^2}$$

$I\ell$ = dipole moment in amp-meters.

As the field point (x,y,z) approaches the surface, Tx goes to zero (since $H\phi$ is zero as previously discussed) and (15) reduces to

$$0 = -\frac{e^{-kr}}{2\pi r}\left(k + \frac{1}{r}\right)\frac{\partial r}{\partial y'}\Bigg|_{\substack{x'=0\\ y'=0\\ z'=0}} + \iint \frac{\partial T_x}{\partial n} \frac{e^{-kr_s}}{2\pi r_s} \, ds \tag{16}$$

where r_s = distance from surface point (x,y,z) to field point, both located on the surface. (Note that the 2′s in the denominator come about by taking the principal value of the integral as the field and source points approach each other on the boundary.) This of course yields a matrix equation of the form

$$[A] \; [\frac{\partial T_x}{\partial n}] = [B] \tag{17}$$

which can be inverted to yield the normal derivatives of T_x on the surface.

By analogy, the y component of T is found as

$$T_y(x,y,z) = \frac{-\partial}{\partial x'} \left(\frac{e^{-kr}}{4\pi r}\right)\Bigg|_{\substack{x'=0\\ y'=0\\ z'=0}} + \oiint \frac{\partial T_y}{\partial n} \, G \, ds \tag{18}$$

and thus, on the surface

$$0 = \frac{e^{-kr}}{2\pi r} \, (k + 1/r) \, \frac{\partial r}{\partial x'}\Bigg|_{\substack{x'=0\\ y'=0\\ z'=0}} + \iint \frac{\partial T_y}{\partial n} \, \frac{e^{-kr_s}}{2\pi r_s} \, ds. \tag{19}$$

We have then a solution for $\frac{\partial T_x}{\partial n}$ and $\frac{\partial T_y}{\partial n}$ on the surface. Once these are known, the solutions for T_x and T_y are easily found in region 1 by simply applying (15) and (18), respectively. It is important to point out that the unknowns $\frac{\partial T_x}{\partial n}$ and $\frac{\partial T_y}{\partial n}$ on the surface vary as a function of sin ϕ and cos ϕ, respectively, on concentric rings centered about the origin in Figure 1. This information greatly reduces the number of unknowns required for solution.

IV. HORIZONTAL DIPOLE - SOLUTION DEVELOPMENT

The horizontal case is truly a good test of the T-Ω method. Unlike the vertical dipole case, two components of T along with Ω are required in region 1. As Carpenter [7] points out, in any truly solenoidal current carrying region ($\nabla \cdot \bar{J} = 0$), two components of $\bar{T}$ are sufficient to describe the current field $\bar{J}$. Sommerfeld [17] argues from the boundary conditions (10 through 13) that a minimum of two components of the magnetic vector potential $\bar{A}$ are necessary for solution. By the T-Ω method, solution is realized through two components of $\bar{T}$ (specifically T_y and T_z) along with Ω (only Ω is needed in air for the quasistatic limit).

The unknowns being sought are T_y, $\frac{\partial T_y}{\partial n}$, T_z, $\frac{\partial T_z}{\partial n}$, Ω_1, $\frac{\partial \Omega}{\partial n}$, Ω_2, and $\frac{\partial \Omega_2}{\partial n}$ all on the surface. The authors again employ symmetry to reduce the number of unknowns. In terms of a polar coordinate ϕ measured from the x axis (see Figure 1), it is found that the unknowns Ω_1, Ω_2, T_z, and their partial derivatives all have a $\sin\phi$ dependence at any fixed radial distance $\sqrt{x^2 + y^2}$ over the surface. The unknowns T_y and $\frac{\partial T_y}{\partial n}$ are constant on any surface ring. The most useful tool in deducing these symmetries is found in analysis of the forcing function $J_s \times \nabla G$. This of course gives information only about $\bar{T}$; symmetries, if any, relevant for Ω are then deduced from the boundary conditions (e.g., $\hat{n} \cdot \|\bar{B}\| = 0 \Rightarrow \Omega$ must have the same spatial dependence as T_z).

Determination of the unknowns results from applying the integral equations for T_z, T_y, Ω_1, and Ω_2 while ensuring a match of the boundary conditions. After the surface is discretized into concentric rings (see Figure 1), variable unknowns are assigned at the inside and outside of each ring. (A linear basis function was chosen for variable variation over each element for most of this work; quadratic basis functions showed no significant improvement in accuracy except at high frequencies.) The following steps summarize the procedure for solution:

1. Because a system redundancy has been introduced (two components of $\bar{T} + \Omega$ versus two components of $\bar{A}$ with Sommerfeld), we are free to arbitrarily affix one of the variable values. Set $\Omega_1 = \Omega_2$ across the surface $(z = h)$.

2. From boundary condition (11) $(\hat{n} \cdot \|\bar{H}\| = 0)$ and the condition $\Omega_1 = \Omega_2$ we deduce that T_y on the boundary is zero. This allows immediate calculation of $\frac{\partial T_y}{\partial n}$ from the integral equation

$$T_y(x,y,z)\Big|_{surface} = 0 = \frac{\partial}{\partial z'} \left(\frac{e^{-kr}}{2\pi r}\right)\Big|_{\substack{x'=0\\ y'=0\\ z'=0}} + \iint \frac{\partial T_y}{\partial n} \left(\frac{e^{-kr}}{2\pi r}\right) ds. \tag{20}$$

3. The remaining unknowns T_z, Ω_1, Ω_2 are represented by their integral equations

$$T_z = \frac{-\partial}{\partial y'} \left(\frac{e^{-kr}}{4\pi r}\right)\Big|_{\substack{x'=0\\ y'=0\\ z'=0}} + \iint \left\{ \frac{\partial T_z}{\partial n} \left(\frac{e^{-kr}}{4\pi r}\right) - T_z \frac{\partial}{\partial n} \left(\frac{e^{-kr}}{4\pi r}\right)\right\} ds$$

or

$$\frac{T_z}{2} = \frac{e^{-kr_o}}{4\pi r_o} (k + \frac{1}{r_o}) \sin\theta \sin\phi + \iint \{\frac{\partial T_z}{\partial n} (\frac{e^{-kr}}{4\pi r})\} \, ds \tag{21}$$

$$\frac{\Omega_1}{2} = \iint \frac{\partial \Omega_1}{\partial n} (\frac{e^{-kr}}{4\pi r}) \, ds \tag{22}$$

$$\frac{\Omega_2}{2} = - \iint \frac{\partial \Omega_2}{\partial n} (\frac{1}{4\pi r}) \, ds \tag{23}$$

where

$$r_o = \sqrt{x^2 + y^2 + z^2}.$$

Note that the factor of 1/2 in equations (21 through 23) is again the result of taking the principle value of the integral as the source and field points approach each other on the boundary.

4. A 3 x 3 matrix of the unknowns T_z, Ω, and $\frac{\partial \Omega_1}{\partial n}$ can be found by combining equations (21 through 23) with the starting assumption, the gauge condition, and boundary condition (12), i.e.,

$$\Omega_1 = \Omega_2 = \Omega \tag{24}$$

$$\nabla \cdot T - k^2\Omega = 0 \Rightarrow [\frac{\partial T_z}{\partial n} = k^2\Omega] \text{ on the surface} \tag{25}$$

$$\hat{n} \cdot \| \bar{B} \| = 0 \Rightarrow \mu(T_z - \frac{\partial \Omega_1}{\partial n}) = \mu_o \frac{\partial \Omega_2}{\partial n}. \tag{26}$$

Final field solution interior to regions 1 or 2 follows as before by application of the basic integral expressions. In region 1, equations (7) and (9) give $\overline{T}$ and Ω in the volume. Once these are known, then the analytical derivative of Ω which must be added to $\overline{T}$ (see equation (1)) can be easily performed. This is seen by recalling that the Green's function G, in the integral, is the only function which depends upon the unprimed variables. Hence, it follows that

$$-\nabla\Omega_1(x,y,z) = \iint \left(\frac{\partial\Omega_1}{\partial n}\nabla G_1 - \Omega_1\nabla\left(\frac{\partial G_1}{\partial n}\right)\right) ds. \tag{27}$$

The same manipulation can be performed on equation (8) for the field calculations in region 2.

V. COMPARISON OF NUMERICAL TESTS WITH KNOWN ANALYTICAL RESULTS

A. Vertical Dipole

The T-Ω BIE (boundary integral equation) formulation has been applied to an electric dipole imbedded in a conducting half space as shown in Figure 1. Figure 2 represents a plot in which $H\phi$ and $E\rho$ are examined as a function of distance. For this plot, the conductivity is 1000 mhos/m with a dipole frequency of 1 Hz. The observation point is located at the same depth as the dipole but is moved laterally away from the source along the x axis. The values shown in Figure 2 (dots) converged to within less than 1 percent of the values predicted by Bannister [3] (solid line).

Lateral wave propagation is also investigated using the T-Ω BIE technique. In the lateral wave regime, the energy travels (1) from the source to the boundary surface, (2) along the surface, and (3) back into the conductor to the point of observation [3,4,9,10]. As the direct effect of the dipole source diminishes with distance due to the high attenuation of a conducting medium, lateral wave propagation becomes the dominant mode of energy propagation. Figure 3 represents fields generated by this phenomenon. To get into this regime, the distance from the source to the observation point must be much greater than a skin depth. Thus, in an effort to decrease the distance required to generate the mode, the frequency is increased to $f = 10^5$ Hz for a $\sigma = 4$ mhos/m. Also, a higher ordered basis function (quadratic) is chosen to help increase the accuracy as well as decrease the number of unknowns. The results predicted by the T-Ω BIE technique (dots in Figure 3) converged to within a few percent of a dB of the values predicted by Bannister [3] and Banos [4] (solid line) as x was extended to 100 meters.

B. Horizontal Dipole

The results of applying the T-Ω BIE technique to a horizontal electric dipole imbedded in a conducting half space are shown in Figures 4 and 5. Figure 4 is a field plot of Hx, Hy, and Hz as a function of the polar angle ϕ.

The observation point is located at the same depth as the dipole source and fixed at a constant radius, ρ. The angle ϕ is then swept from 10 degrees to 90 degrees. The results are again within 1 percent of Bannister's [3] results.

Figure 5 has the observation point located in air (at $\phi = 90°$). For this case, the $\sigma = 4$ mhos/m and $f = 1000$ Hz. The results are compared with a formulation generated by intergrating Sommerfeld's integrals numerically (Wynn [18]). The results are within less than 1 percent of Wynn's results except at $R = 20$ where a 1.5 percent deviation was noted (appeared to be due to Rlimit too small).

VI. CONCLUSIONS

The test results validate the accuracy of the T-Ω method for unbounded problems. Of particular interest to potential researchers using this method in a truly unbounded problem should be the search for symmetries that might help reduce the number of unknowns. For an assymetrical problem or high frequency lateral wave problem, more sophisticated basis functions must be investigated in order to improve accuracy as well as reduce the number of unknowns required for solution. The authors are presently examining such techniques in the analysis of a finitely conducting wedge.

VII. REFERENCES

1. P. Balling, "On the Role of Lateral Waves in the Radiation from the Dielectric Wedge," IEEE Trans. on Antennas and Propagat., pp. 247-248, March 1973.

2. P. R. Bannister, "The Quasi-Near Fields of Dipole Antennas," IEEE Trans. on Antennas and Propagat., Vol. AP-15, No. 5, pp. 618-626, September 1967.

3. P. R. Bannister, et al., Quasi-Static Electromagnetic Fields, Naval Underwater Systems Center, 1979.

4. A. Banos, Dipole Radiation in the Presence of a Conducting Half Space, Pergamon Press, 1966.

5. J. Barnes and K. Davey, "Boundary Integral Formulation of Fields Generated Sources Imbedded in Conducting Media," IEEE Southeastcon, Destin, FL, pp. 88-91, April 5-9, 1982.

6. C. Carpenter, "Finite Element Models and Their Application to Eddy Current Problems," Proc. IEEE, Vol. 122, No. 4, pp. 455-462, April 1975.

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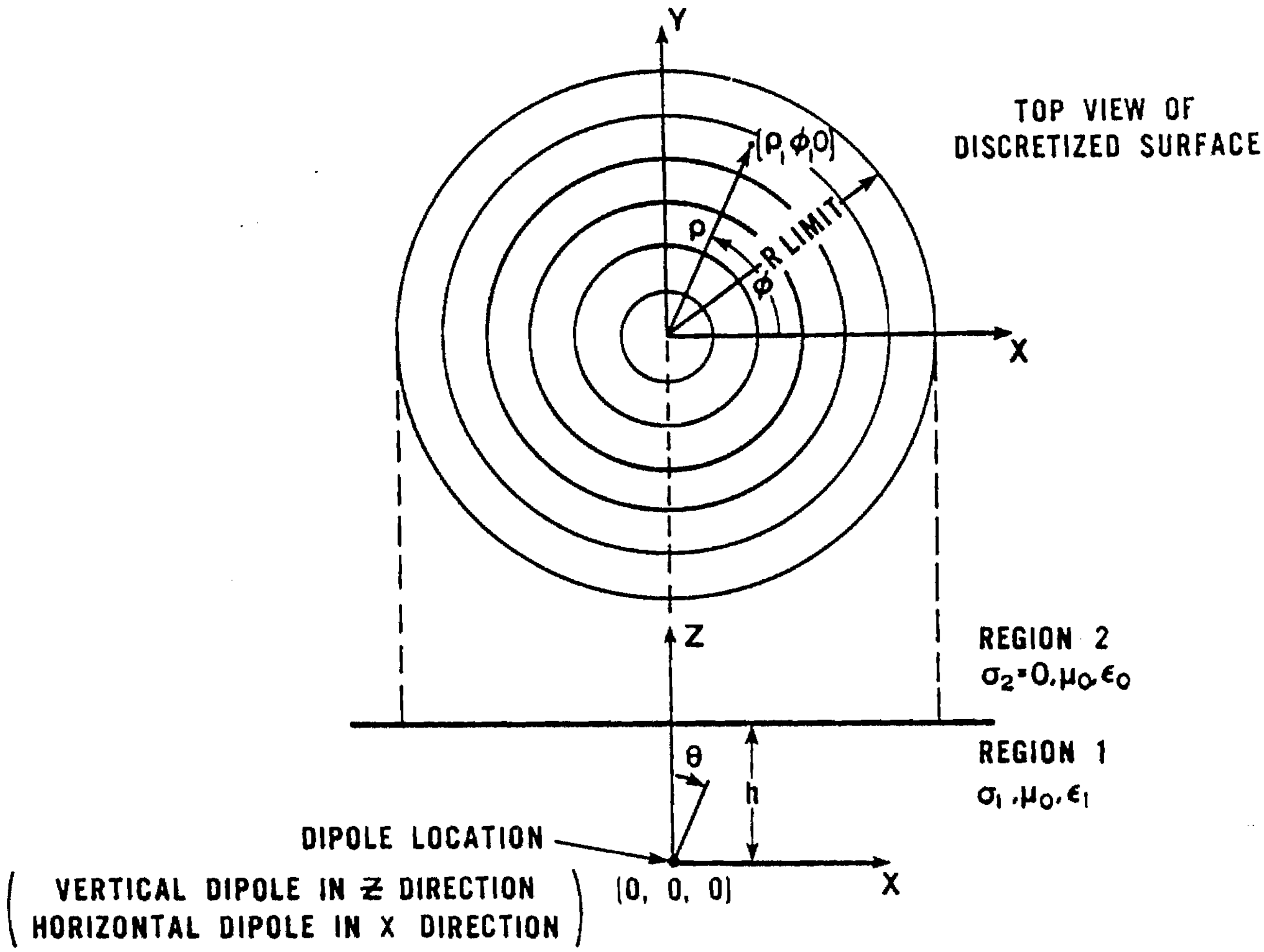

FIGURE 1. ELECTRIC DIPOLE LOCATED IN A CONDUCTING HALF-SPACE WITH COORDINATE SYSTEM AND PARAMETER DEFINITIONS

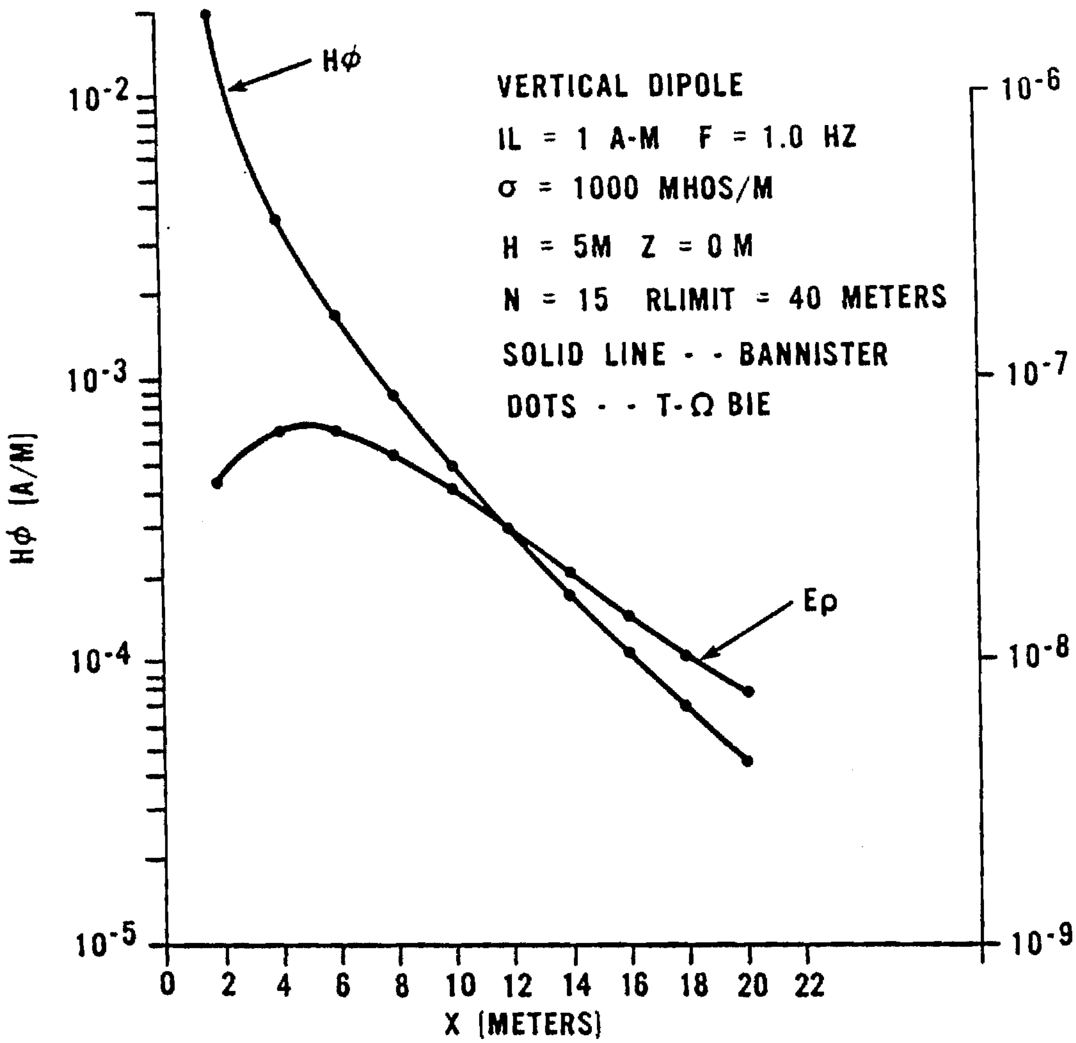

FIGURE 2. E AND H FIELDS AS A FUNCTION OF X FOR A VERTICAL ELECTRIC DIPOLE WHERE BOTH THE SOURCE AND OBSERVATION POINTS ARE LOCATED IN THE CONDUCTING MEDIUM

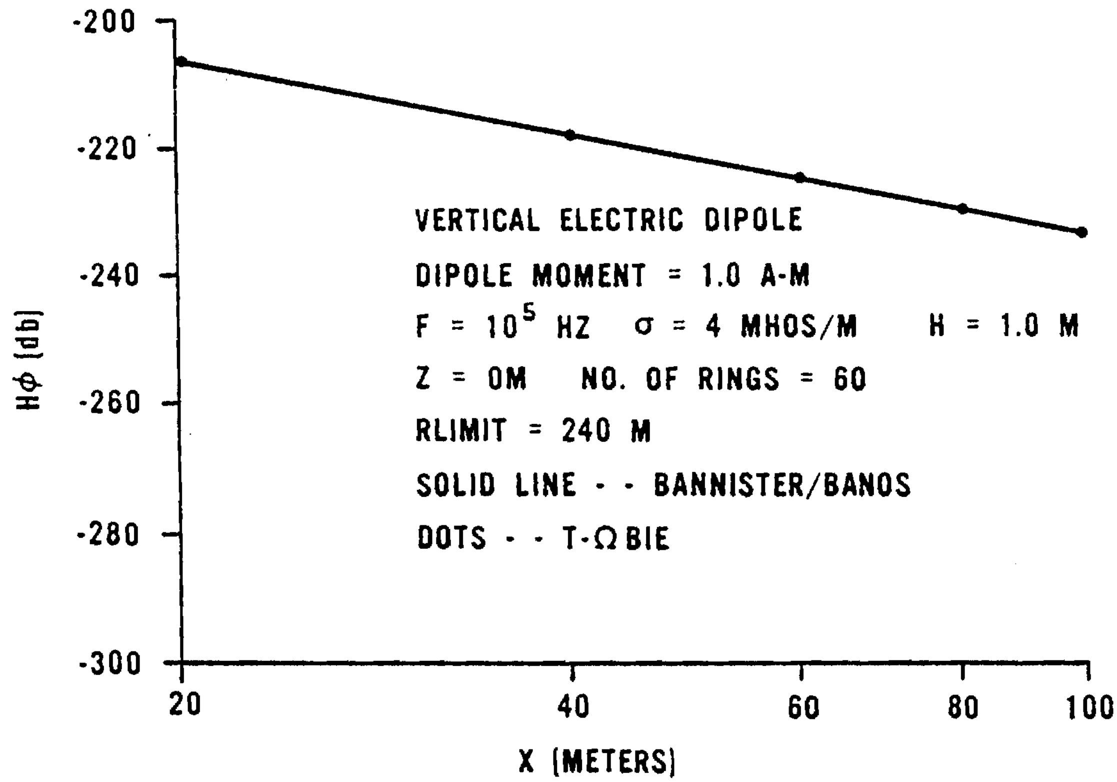

FIGURE 3. Hϕ AS A FUNCTION OF X FOR A VERTICAL ELECTRIC DIPOLE WHERE BOTH THE SOURCE AND OBSERVATION POINTS ARE LOCATED IN THE CONDUCTING MEDIUM. LATERAL WAVE PROPAGATION IS THE DOMINANT MODE

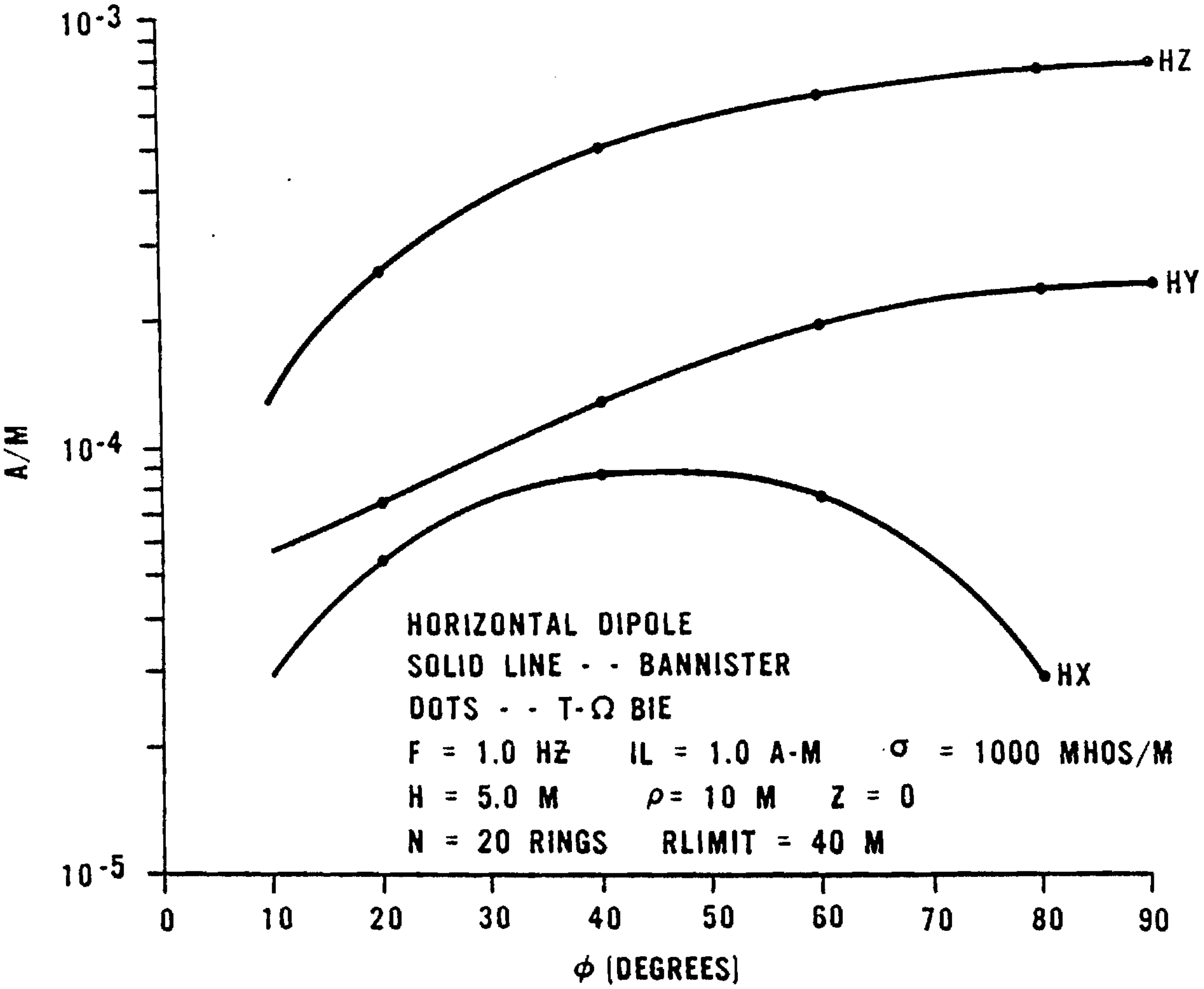

FIGURE 4. H FIELD AS A FUNCTION OF POLAR ANGLE FOR A HORIZONTAL ELECTRIC DIPOLE WHERE BOTH THE SOURCE AND OBSERVATION POINTS ARE LOCATED IN THE CONDUCTING MEDIUM

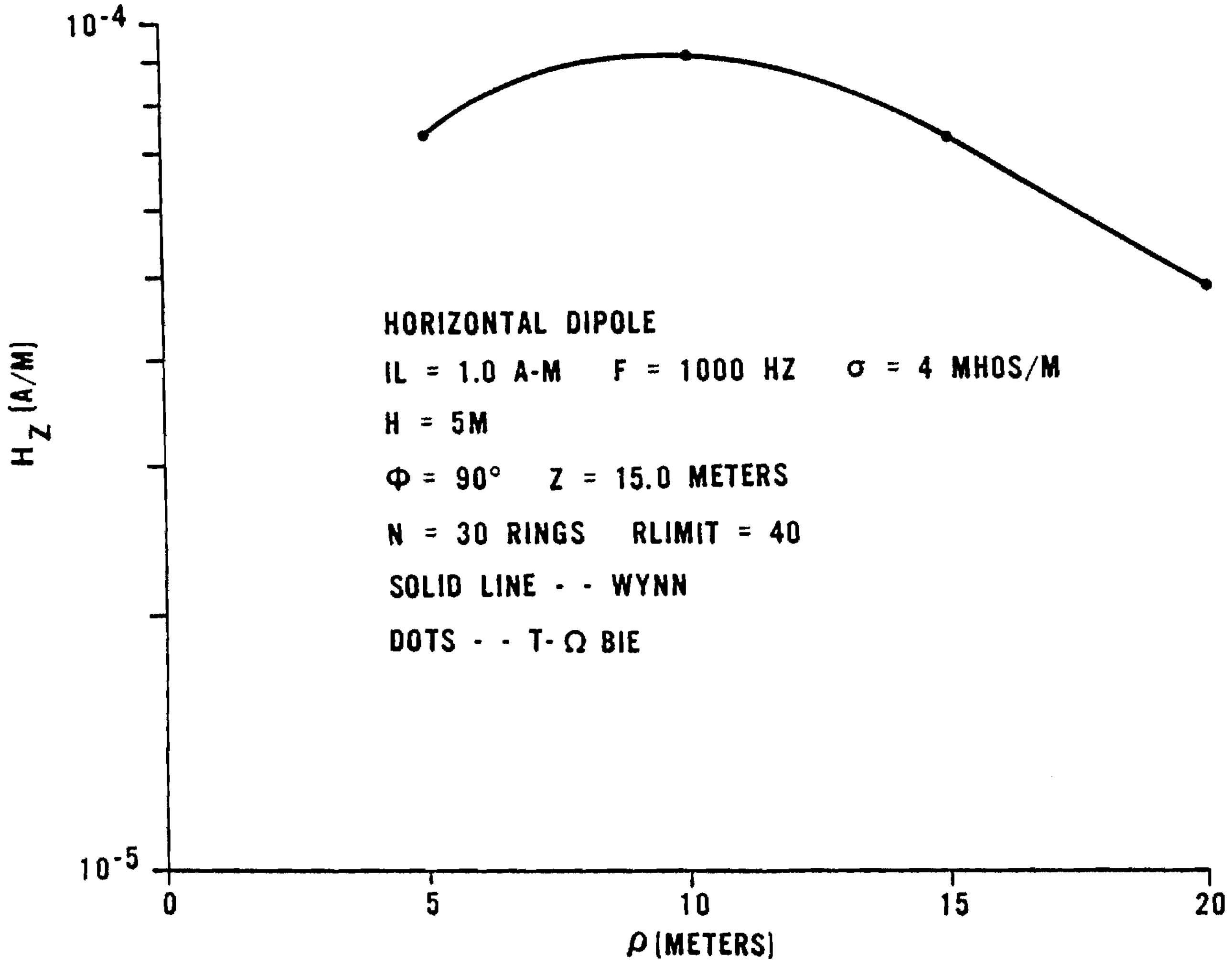

FIGURE 5. H_Z AS A FUNCTION OF ρ FOR A HORIZONTAL ELECTRIC DIPOLE WHERE THE SOURCE AND OBSERVATION POINTS ARE LOCATED IN THE CONDUCTING MEDIUM AND AIR, RESPECTIVELY